# 基于模糊裁剪阈值的 SAMP 压缩感知算法

郭青青,周飞飞,李　雷
(南京邮电大学 理学院,江苏 南京 210023)

**摘　要**:稀疏度自适应匹配追踪(SAMP)算法是压缩感知(CS)中一种主流的图像重构算法。随着迭代次数的增加,SAMP 算法的原子候选集将成倍增加,会导致系统空间的浪费和重构时间的增长。为此,提出了一种模糊裁剪阈值稀疏度自适应匹配追踪(FPTSAMP)算法。由于离散小波变换(DWT)在 CS 稀疏处理过程中破坏了低频逼近系数间的相关性,对信号的重构质量产生了一定的负面影响,因而采用小波高频子带变换(HFSBWT)来替代 DWT,实现对信号的稀疏表示。仿真实验结果表明,相比于同一重构算法,采用 HFSBWT 方法得到的峰值信噪比更好;与 SAMP 算法相比,与 HFSBWT 相结合的 FPTSAMP 算法的重构效果有了明显提高,重构时间也减少了一半。

**关键词**:压缩感知;重构算法;高频子带小波变换;模糊裁剪阈值 SAMP 算法
**中图分类号**:TP301.6　　**文献标识码**:A　　**文章编号**:1673-629X(2017)09-0035-05
doi:10.3969/j.issn.1673-629X.2017.09.008

# Sparsity Adaptive Matching Pursuit Algorithm for Compressed Sensing with Fuzzy Pruning Threshold

GUO Qing-qing,ZHOU Fei-fei,LI Lei
(Faculty of Science,Nanjing University of Posts and Telecommunications,Nanjing 210023,China)

**Abstract**:Sparsity Adaptive Matching Pursuit (SAMP) is a mainstream image reconstruction algorithm in Compressed Sensing (CS). However,with the increase of iterative times,it has multiplied atoms candidate set that lead to wasting storage capacities and lengthening reconstruction time. A method called Fuzzy Pruning Threshold Sparsity Adaptive Matching Pursuit (FPTSAMP) is proposed. The Discrete Wavelet Transform (DWT) destroys the correlation among low-frequency approximation coefficients in CS sparsity processing, which results in bad reconstruction quality,so a High Frequency Sub-Band Wavelet Transform (HFSBWT) is adopted instead of DWT to realize the sparse representation of signal. Simulation results show that compared with the same reconstruction algorithms the HFSBWT has achieved a better Peak Signal To Noise Ratio (PSNR) of images and that compared with SAMP algorithm the FPTSAMP combined with HFSBWT has lifted the reconstruction performance of images significantly with its reconstruction time cutting in half.

**Key words**:compressed sensing;reconstruction algorithm;high frequency sub-band wavelet transform;fuzzy pruning threshold sparsity adaptive matching pursuit

## 1　Summarization

Compressed Sensing (CS)[1-3] theory had broken through the restriction of Nyquist sampling theorem on the sampling rate, and realized sampling and compressing data simultaneously. CS projects high-dimensional data to low-dimensional data, and recovers the signal by projection observation and reconstruction algorithm. It is described as follows:

$$\boldsymbol{y} = \boldsymbol{\Phi x} = \boldsymbol{\Phi\Psi\theta} = \boldsymbol{A\theta} \tag{1}$$

Where $\boldsymbol{x} \in R^N$ is the $K$-sparse signal which can be represented sparsely in a certain transform domain $\boldsymbol{\Psi}$, i.e. $\boldsymbol{x} = \boldsymbol{\Psi\theta}$, $\boldsymbol{y}$ denotes the measurement values vector with reduced dimension $n$, $\boldsymbol{\Phi}$ is a measurement matrix with size $n \times N$, and $\boldsymbol{A} = \boldsymbol{\Phi\Psi}$ is called a $n \times N$ sensing matrix.

The signal can be reconstructed from relatively few incomplete measurements $\boldsymbol{b} = \boldsymbol{Ax}$ for a carefully chosen

**收稿日期**:2016-06-15　　**修回日期**:2016-09-22　　**网络出版时间**:2017-07-05
**基金项目**:国家自然科学基金资助项目(61501251,61071167,61373137);江苏省普通高校研究生科研创新计划资助项目(KYZZ15_0236);南京邮电大学引进人才科研启动基金资助项目(NY214191)
**作者简介**:郭青青(1991-),女,硕士,研究方向为信号图像处理、数据可视化和数值计算;李　雷,教授,研究方向为模式识别和智能系统、计算机网络、非线性分析、计算与应用数学、数学和模糊系统以及信号处理等。
**网络出版地址**:http://kns.cnki.net/kcms/detail/61.1450.TP.20170705.1649.018.html

$A \in R^{m \times n}$ by solving the $l_0$ –minimization problem.

$$\min_{\theta} \ \| \hat{\theta} \|_{l_0} \quad \text{s. t.}\ A\hat{\theta} = \Phi \Psi \hat{\theta} = y \tag{2}$$

Where $\| \cdot \|_{l_0}$ denotes $l_0$–norm, and $\hat{\theta}$ represents the approximation vector of the sampled sparse vector $\theta$ . Unfortunately, it is not convex and the computational complexity of optimizing is NP hard. Then, Donoho and Tao etc. have proposed that the solution $x$ of the problem(1) can be found by solving the Basis Pursuit (BP) problem, under some reasonable conditions on $x$ and $A$ .

$$\min_{\theta} \ \| \hat{\theta} \|_{l_1} \quad \text{s. t.}\ A\hat{\theta} = \Phi \Psi \hat{\theta} = y \tag{3}$$

Where $\| \cdot \|_{l_1}$ denotes $l_1$–norm, if $\Phi$ and $\Psi$ are incoherent and sensing matrix $A$ satisfies the Restricted Isometry Property (RIP), then the $K$ –sparse signal $x$ can be well reconstructed. Thus, the non–convex problem is transformed into a convex one to solve.

$l_1$ – linear programming problem has been found several algorithms including BP[4], Gradient Projection for Sparse Reconstruction (GPSR)[5], and Iterative Thresholding (IT)[6]. For the $l_0$ – optimization problem, iterative greedy pursuit is a good idea that contains Orthogonal Matching Pursuit (OMP)[7], Compressive Sampling Matching Pursuit (CoSaMP)[8], Stagewise Orthogonal Matching Pursuit (StOMP)[9] and Sparsity Adaptive Matching Pursuit (SAMP)[10], etc.

The CS theory has three main steps: signal sparse representation, measurement matrix and reconstruction algorithm.

Signal sparse representation is premise of compressed sensing technique to ensureits effectiveness. In this paper, High – Frequency Sub – Band Wavelet Transform (HFSBWT) sparse representation has been provided instead of traditional image sparsity, wavelet basis or Discrete Wavelet Transform (DWT)[11] to enhance sparsity property and to promote reconstruction effect of images.

The most critical step for compressed sensing is reconstruction, which determines performance and operation time of reconstruction. To deal with wasting storage capacity and time–consuming operation with Sparsity Adaptive Matching Pursuit (SAMP) algorithm, Fuzzy Pruning Threshold Sparsity Adaptive Matching Pursuit (FPTSAMP) algorithm has been proposed, which adopts fuzzy threshold preliminary rule to avoid using a priori information of signals in primary election phase and then realizes adaptive recovery and achieves purpose of shrinking the atoms selection space and cutting down iteration time.

In this paper, a novel transform named HFSBWThas been adopted to improve performance of traditional transform and then FPTSAMP algorithm has been proposed to deal with wasting space and time–consuming on SAMP algorithm, which introduces the fuzzy preselected method, and pruning and stop threshold to eliminate redundant atom and reduce unnecessary iterations. The proposed algorithm has not only improved reconstruction precision but also accelerated convergence speed. In addition, some simulation experiments has been conducted to verify better performance of FPTSAMP algorithm over others.

## 2　Improved FPTSAMP Reconstruction Algorithm Based on HFSBWT Sparse Representation

### 2.1　The HFSBWT Sparse Representation

The first step of CS Theory is to transform the original non–sparse signal to sparse signal. To make the image signal sparse, universally the method of sparse transformation DWT[3-4] is adopted. After having decomposed image with single–layer wavelet transform, four sub–band coefficient matrixes is acquired including average part, vertical detail, horizontal detail and diagonal detail, i. e. $\{c_A, c_V, c_H, c_D\}$ , where average part should be deemed to frequency component of image, and the remaining three parts are considered as high frequency section. Low–frequency part can be regarded as an approach signal of the original image within the different scales, so the low–frequency component is non–sparse and difficult to be more sparse. Nevertheless, the remaining three high frequency parts are sparse relatively. If coefficients of the high–frequency and low–frequency sub–band simultaneously multiplied with measurement matrix to obtain the measured values the correlation among the low–frequency approximation coefficients would be destroyed, which could lead to a worse quality of the reconstructed signal.

So anovel kind of image sparse transform way is provided based on compressed sensing, called HDSBWT[12-13]. First, the product of $\Phi$ and $c_V$ , $c_H$ , $c_D$ is calculated respectively with high frequency sub–band measurements $c_v$ , $c_h$ , $c_d$ , while the low–frequency sub–band is remained in $c_A$ , then setting a measuring number $M$ , and constructing i. i. d. Gaussian matrix as measurement matrix $\Phi$ with the size of $M \times Q$ , where $Q \approx N/2$, furthermore in decoder side, reconstruction algorithm was used to recovery three high frequency coefficient matrixes $c_V$ , $c_H$ , $c_D$ respectively. Finally inverse wavelet transform together with sub-band $c_A$ was carried out to reconstruct image.

## 2.2 Improved FPTSAMP Reconstruction Algorithm

SAMP[14] is a kind of typical algorithm to solve the optimization problem of $l_0$ - norm with two advantages of adaptive sparsity and involving retrospective thoughts. However, there are mainly two drawbacks that appeared during pre-selection and pruning atom:

(1) Atpre-selection stage, when $k$ is larger, index of new added atom is seldom due to the weak-correlation between the residual value $r_{k-1}$ and the sensing matrix. Then the excessive collection of atom candidate index leads to choosing atom support set repeatedly, even causes large consuming of time, and impacts accuracy of least squares estimation method.

(2) At pruning atom stage, the iteration stepwould be doubled while there is a failure to meet the constraint, so that number of elements in candidate set would be $S$ times that of original set at each stage. While candidate set are superfluous, it is bound to affect the composition of support set.

Aimed at the drawback of SAMP algorithm, an improved algorithm named FPTSAMP has been proposed. Where fuzzy pre-selection stage is set and the original candidate atom method is replaced by adding pruning threshold.

Having been inspired by stagewise weak threshold conjugate gradient algorithm and subspace pursuit algorithm, FCTSAMP adopts fuzzy threshold instead of the original pre-selection program.

$$\begin{cases}\mathrm{Th} = \alpha_{\mathrm{pr}} + \mathrm{rand}(1) * (\beta_{\mathrm{pr}} - \alpha_{\mathrm{pr}}) \\ S = \{ i \mid \mathrm{idx}(\mathrm{sort}(\mathrm{Th} * \mathrm{abs}(\boldsymbol{A}^{\mathrm{T}} r_{k-1}), \text{'decend'})) \}\end{cases} \tag{4}$$

Where sort( · ,'decend') is descending order elements, and idx( · ) is extracting element index, $\alpha_{\mathrm{pr}}$ and $\beta_{\mathrm{pr}}$ are two fuzzy threshold parameters available for the user to set, and function rand(1) can generate random numbers range from 0 to 1. Because of each iteration the weak-correlation of $\boldsymbol{A}$ and $r_{k-1}$ leads to uncertain number of new added subscript. Compared with the original preselected program the fuzzy threshold method promotes the correlation to manipulate more new subscript with common sense.

For the problem of repeated prune with SAMP in each of iteration, FPTSAMP sets stop and pruning thresholds at the first time. When residual energy norm is less than the stop threshold $p_1 * \|\boldsymbol{y}\|_2$, which indicates that the current residual of iteration is small enough, no cutting operation is needed, or demonstration meets iteration termination conditions to avoid unnecessary cutting operation and to save calculations time. When residual energy norm is less than the pruning threshold $p_2 * \|\boldsymbol{y}\|_2$, this situation suggests pruning operation of atomic candidate set need to be implemented. To crop the redundant atoms accurately one subscript in candidate set at a time has been discarded and then it is determined whether change the residual value $r$ or not. If the residual value $r$ increases, it indicates that no need to perform pruning operation, the step size of candidate set has been doubled, then access to the next iteration. Otherwise, if the residual value is smaller or unchanged, indicating that the subscript being pruned is certainly redundant, then iterating the pruning step until redundancy is eliminated.

As mentioned above, there are four input parameters, $\alpha_{\mathrm{pr}}$ , $\beta_{\mathrm{pr}}$ , $p_1$ and $p_2$. Based on huge simulation experiments and balance between operation time and reconstruction accuracy, if setting $\alpha_{\mathrm{pr}} = 0.8$, $\beta_{\mathrm{pr}} = 1$, and selecting $p_1 = 6 \times 10^{-4}$ , $p_2 = 1 \times 10^{-2}$ , performance of signal reconstruction is better. In brief, the FPTSAMP execution process lists as follows:

(1) Input: sensing matrix $\boldsymbol{A}$ , observation vector $\boldsymbol{y}$ , fuzzy threshold $\alpha_{\mathrm{pr}}, \beta_{\mathrm{pr}}$ , stopping threshold $p_1$, pruning threshold $p_2$, step factor $S$ , tolerance error $\delta$ .

(2) Initialization: approximation signal $\hat{x} = 0$, residual value $r_0 = y$ , support set $F_0 = \varnothing$ , size of support set $L = S$ , let $n = 1$.

(3) For $k = 1$, $k = k + 1$ until meeting stopping criterion $\|r_k\|_2 \geqslant \delta$

(a) $J = \mathrm{abs}(\boldsymbol{A}^{\mathrm{T}} * r_{k-1})$ , $\mathrm{Th} = \alpha_{\mathrm{pr}} + \mathrm{rand}(1) * (\beta_{\mathrm{pr}} - \alpha_{\mathrm{pr}})$ .

(b) $H = \mathrm{abs}(\mathrm{Th} * J)$ , $H = \mathrm{sort}(H, \text{'decend'})$ .

(c) $S_k = \{ i \mid i = \mathrm{idx}(H(t)), 1 \leqslant t \leqslant L \}$, $C_k = F_{k-1} \cup S_k$ .

(d) If( $\|r_{k-1}\|_2 \leqslant p_1 * \|\boldsymbol{y}\|_2$ ), break; else go on.

(e) If( $\|r_{k-1}\|_2 \leqslant p_2 * \|\boldsymbol{y}\|_2$ ), $G = A_{C_k}^{\dagger} * y$, $G = \mathrm{sort}(G, \text{'decend'})$, $F = \{ j \mid j = \mathrm{idx}(G(t)), 1 \leqslant t \leqslant L - k \}$ , $r_{\mathrm{res}} = y - A_F A_F^{\dagger} y$ ; if( $\|r_{k-1}\|_2 \geqslant \|r_{\mathrm{res}}\|_2$ ), $n = n + 1$, $L = n * L$ ; else $C_k = F$, $r_{k-1} = r_{\mathrm{res}}$ , return back to sub

-step (e).

(f) $r_{k+1} = y - A_{C_k} A_{C_k}^{\dagger} * y, F_k = F$.

(4) $\hat{x}_{F_k} = A_{F_k}^{\dagger} * y$.

(5) Output: approximation of original signal $\hat{x} = \hat{x}_{F_k}$.

## 3 Experimental Results and Analysis

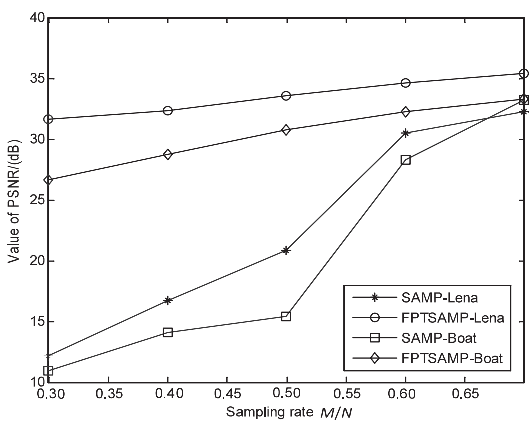
In order to verify the advantage of reconstruction accuracy and time with FPTSAMP over those with SAMP, in experiments, HFSBWT transform, sampling $256 \times 256$ Lena and Boat images respectively at rate $M/N = 0.3$, then image sparsity handled by HFSBWT transform, and measurement matrix is i. i. d. Gaussian matrix. Finally, reconstructed effect of Boat and Lena images have been completed by the FPTSAMP and SAMP respectively, shown by Fig. 1.

(a)Original Lena image　(b)Reconstructed Lena image based on SAMP　(c)Reconstructed Lena image based on FPTSAMP

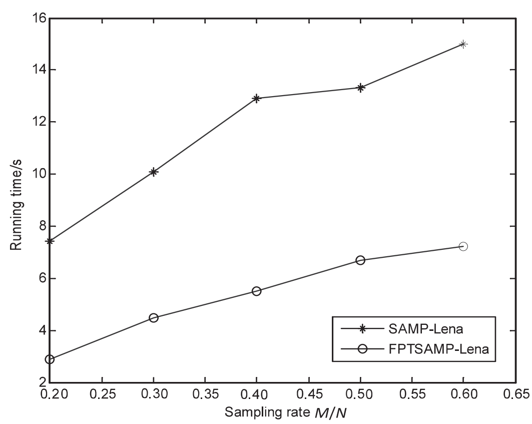
(d)Original Boat image　(e)Reconstructed Boat image based on SAMP　(f)Reconstructed Boat image based on FPTSAMP

Fig. 1　Reconstruction effect comparison of images

Contrast analyses of Fig. 1 shows that when sampling rate is 0.3, the reconstructed image achieved by SAMP reconstruction algorithm may have a significant fuzzy effect and there is a lot of interference when regardless of Lena or Boat image. However the reconstructed image achieved by FPTSAMP reconstruction algorithm, its texture pattern is visible clearly, only fuzzier than original image slightly. In order to illustrate the superior performance of FPTSAMP, more effective comparison with SAMP and comparison charts in Fig. 2.




(a)PSNR




(b)Running time

Fig. 2　Comparing reconstructed performance of SAMP and FPTSAMP

As can be seen from Fig. 2(a), when at the lower sampling rate, the reconstruction PSNR of two algorithms are relatively low, but with sampling rate increasing, the PSNR also improves significantly. And PSNR achieved by FPTSAMP is always higher than that achieved by SAMP. So the improved algorithm is much better than the original algorithm with advantageous reconstruction performance. Because of fuzzy pruning threshold taken into account in the improved algorithm, each iteration time could be shortened and convergence rate be sped up. Above all, Lena image for example, draw running time contrastive figure of two algorithms under various sampling rates shown in Fig. 2(b).

With increase of the sampling rate Fig. 2(b) shows that running time increases, but the increasing amplitude in FPTSAMP is always shorter than that in SAMP. Moreover, in terms of various sampling rates, the running time of FPTSAMP is significantly shorter than that of the SAMP so that time almost cut by half.

## 4 Conclusion

Based on the classical pursuit algorithm SAMP, FPTSAMP has been proposed, which introduces the fuzzy preselected method, pruning threshold and stop threshold to cut down redundant atom, to reduce unnecessary iterations, to improve reconstruction precision, and to accelerate convergence speed. In addition, the traditional DWT transform breaks the correlation among coefficients and leads to a poor reconstruction performance, the HFSBWT has been proposed. A large number of simulation experiments show that after having handled images with sparse transform HFSBWT and employed FPTSAMP for recovery images, the reconstruction performance has been improved significantly and its running time is cut by half.

**参考文献:**

[1] Donoho D L. Compressed sensing[J]. IEEE Transactions on Information Theory,2006,52(4):1289-1306.

[2] Candès E J,Wakin M B. An introduction to compressive sampling[J]. IEEE Signal Processing Magazine,2008,25(2):21-30.

[3] Gao Z,Xiong C, Ding L,et al. Image representation using block compressive sensing for compression applications[J]. Journal of Visual Communication and Image Representation,2013,24(7):885-894.

[4] Chen S S,Donoho D L,Saunders M A. Atomic decomposition by basis pursuit[J]. SIAM Review,2001,43(1):129-159.

[5] 徐　健,常志国,赵小强,等. 基于自适应梯度最速下降的分类字典训练算法[J]. 计算机工程,2013,39(5):225-229.

[6] 张宗念,李金徽,黄仁泰. 迭代硬阈值压缩感知重构算法——IIHT[J]. 计算机应用,2011,31(8):2123-2125.

[7] Donoho D L,Tsaig Y,Drori I,et al. Sparse solution of underdetermined systems of linear equations by stagewise orthogonal matching pursuit[J]. IEEE Transactions on Information Theory,2012,58(2):1094-1121.

[8] Needell D,Tropp J A. CoSaMP:iterative signal recovery from incomplete and inaccurate samples[J]. Communications of the ACM,2010,53(12):93-100.

[9] 周　燕,曾凡智,赵慧民,等. 一种基于精细化稀疏自适应匹配追踪算法的图像检索方法研究[J]. 电子学报,2014,42(12):2457-2466.

[10] 杨　成,冯　巍,冯　辉,等. 一种压缩采样中的稀疏度自适应子空间追踪算法[J]. 电子学报,2010,38(8):1914-1917.

[11] Zhang Y,Mei S,Chen Q,et al. A novel image/video coding method based on compressed sensing theory[C]//IEEE international conference on acoustics,speech and signal processing. [s. l. ]:IEEE,2008:1361-1364.

[12] 甘　伟,许录平,罗　楠,等. 一种自适应压缩感知重构算法[J]. 系统工程与电子技术,2011,33(9):1948-1953.

[13] 周飞飞,李　雷. 小波高频子带变换裁剪阈值 SAMP 算法研究[J]. 计算机技术与发展,2014,24(5):83-86.

[14] 甘　伟,许录平,张　华,等. 一种贪婪自适应压缩感知重构[J]. 西安电子科技大学学报:自然科学版,2012,39(3):50-57.